An Approximation of the Integrated Infrared Starlight at 2.2 and 3.6 μ over the Sky

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The integrated starlight for the whole sky has been computed for Johnson's *K* and *L* bands, centered at 2.2 and 3.6 μ, respectively, from Charlier's compilation of the "Henry Draper Catalogue." Isophotes for the *K* and *L* bands are shown on Aitoff diagrams for both galactic and equatorial coordinates. A qualitative analysis of the error of these computations is offered.

Introduction

In broad-band photometric studies of the night airglow and auroras, it is necessary to subtract from the observed intensity the intensity of the astronomical background. In this article we are concerned only with the infrared background. A rough analysis of the relative intensities of the various components of the astronomical background indicates, as it might be expected, that the stellar component is probably singularly significant. The only exception might be interstellar regions of neutral and ionized hydrogen. However, the figures for the intensity emitted by hydrogen regions are probably greatly in excess because of the lack of adequate theory for their emission mechanism and for the opacity of this material. A summary of the astronomical background is presented in Table 1. For the purpose of visualization, an Echo-type satellite at 300 km has been taken as the comparison intensity.

Results obtained herein and the observations of Moroz[3] of the night airglow in the 1- -3.4-μ range, and of Rodionov and Fishkova[4] of the very near infrared auroras permit a rough general estimate of the significance of the infrared stellar background in the 1- -3.4-μ region. The total intensity in w-cm^{-2}-sr^{-1} of the airglow, auroras, and troposphere emission are roughly 10^{-9}, 10^{-9}, and 10^{-8}, respectively, as compared to an estimate of 10^{-6} for the stellar background in the 1- -3.4-μ region. The Soviet photometric observations of the night *OH* band spectrum were taken within sufficiently narrow bandwidth so as not to contain a significant stellar component. The 10^{-6} figure for the stellar intensity is an average of the entire sky.

Roach and Megill[5] have computed isophotes of stellar sky in two astronomical bandwidths in the visible region of the spectrum to provide an estimate for the stellar component of broad-band photometric observations of the night airglow. To this same end additional isophotes for Johnson's[6] bandwidths at 2.2 and 3.6 μ are computed herein to provide an estimate of the infrared stellar background. These isophotes are given in units of astronomical magnitudes per square degree. A rough estimate for their conversion to intensity in w-cm^{-2}/deg^2 is given. A more accurate conversion must await a detailed knowledge of Johnson's photometer response function. However, with our rough values for the intensities in the *K*, 2.2-μ, and *L*, 3.6-μ bands, it is possible to obtain an estimate of the stellar intensity in an arbitrary band. The intensity in an arbitrary band follows directly from the monochromatic stellar distribution in a given region of the sky. The monochromatic distribution may be assumed to be linear or, say, that of a backbody distribution fitted to the *K* and *L* intensities reduced to their effective wavelengths.

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Available Stellar Data, Charlier's Compilation

This computation of the infrared integrated starlight requires a star compilation in the form of the number of stars of a given range of spectrum-luminosity type (e.g., between *A0V* and *A9V*), having a given interval of apparent magnitude ($m - \frac{1}{2}\Delta m < m \leq m + \frac{1}{2}\Delta m$) and lying within a given area on the celestial sphere (of solid angle Ω). This number will be denoted by $N = N_{s,m}(l,b)$, where l is the galactic longitude and b the galactic latitude of the "center" of each area. The only compilation of this type covering the entire sky is that of Charlier[7, 8] which is complete to at least the eighth visual magnitude (9.5 in some regions).[9] Charlier prepared his compilation from the "Henry Draper Catalogue,"[10] which contains approximately 225,000 stars. The "Bergedorfer Spektral-Durchmusterung"[11] contains tabulation of N complete to the twelfth photographic magnitude, but this compilation is for small selected areas (the Kapteyn areas) scattered uniformly throughout the northern sky. Next to Charlier's compilation, that of Shapley and Cannon[10] is most suitable for our purpose.

Charlier divided the celestial sphere into 48 equal areas as shown in Fig. 1. The approximate position of a number of constellations is also shown in Fig. 1. The area of each square is $4\pi/48$ sr or 859.44 deg^2. Within each square Charlier counted the number of stars N having apparent

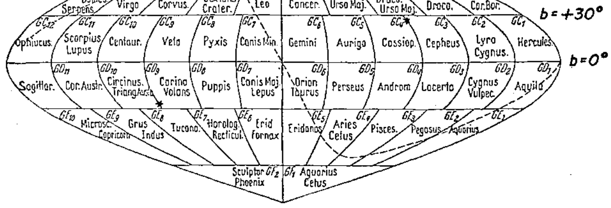

Fig. 1 Charlier's 48 equal areas reproduced with the approximate location of constellations and Chalier's designation for each area; the Galactic coordinates are referred to the Ohlsson Galactic North Pole (i.e., at α = 190° δ = +28°).

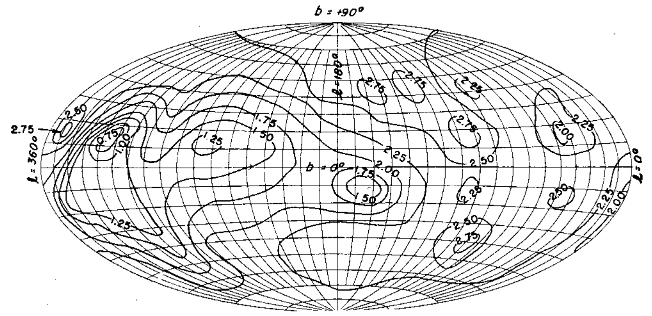

Fig. 2 Isophotes for the stellar sky in the *K*-band (2.2 μ) of Johnson; contour values are in magnitudes per square degree superimposed on Galactic coordinates.

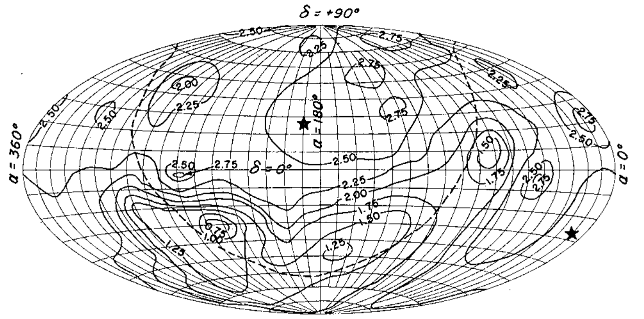

Fig. 4 Isophotes for the stellar sky in the *K*-band (2.2 μ) of Johnson; contour values are in magnitudes per square degree superimposed on equatorial coordinates.

visual magnitude intervals $-1 \pm 0.5, 0 \pm 0.5, 1 \pm 0.5, \ldots, 12 \pm 0.5$, for each of the following spectral groups: *O*, *B–B5*, *B8–B9*, *A*, *F*, *G*, *K*, *M*, and *N*.

Charlier's values N_C (where the subscript C denotes Charlier's number) have been further divided, for *G*-type stars and later, into two luminosity groups containing luminosities I–III and IV–V (for reasons that will be apparent later) after the luminosity classification of Morgan, Keenan, and Kellman.[12] The division adopted has been taken from Allen (Ref. 2, p. 217). Table 2 gives the fraction P_s and $1 - P_s$, respectively, for the stars of various spectral types in each of our two groups I–III and IV–V. The fractions in Table 2 are representative of the mean value for the whole sky. No attempt will be made here to obtain P_s as a function of l and b, and, of course, P_s will be assumed to be independent of m. Therefore, if N_C values are Charlier's numbers, then the number of stars in an area of the sky centered at (l,b) within spectral type interval S and magnitude interval $m \pm 0.5$ within luminosity groups I–III and IV–V, are, respectively,

$$N_C' = P_s N_C \qquad N_C'' = (1 - P_s) N_C \tag{1}$$

and therefore our adopted compilation is

$$N_A = \begin{cases} N_C \text{ for spectral groups } O, B\text{–}B5, B8\text{–}B9, A, \text{ and } F \\ N_C' \text{ for spectral groups } G, K, M, \text{ and } N \text{ in luminosity group I–III} \\ N_C'' \text{ for spectral groups } G, K, M, \text{ and } N \text{ in luminosity group IV–V} \end{cases} \tag{2}$$

Description of Computational Procedure

Having established N_C' and N_C'' for each of Charlier's 48 areas, we now compute the integrated visual magnitude, within each area, for each of our spectrum-luminosity groups, which are 13 in number (i.e., *O*, *B–B5*, *B8–B9*, *A*, *F*, *G*I–III, *G*IV–V, *K*I–III, *K*IV–V, *M*I–III, *M*IV–V, *N*I–III, and *N*IV–V). This effectively reduces the multitude of stars in each area to 13 representative stars. (The accuracy of this representation, in principle, should improve as the spectral and magnitude intervals are lessened.) These 13 stars will be thought of as possessing the spectral type of the midspectral interval, e.g., *F*5 will be assigned as the effective spectral type for the star obtained from the integrated light in a given area for Charlier's *F* group. The magnitude of 13 representative stars in each of the 48 areas will each be converted to an infrared magnitude (in the *K*- and *L*-bands) by means of *K-V* and *L-V* color indexes compiled from Johnson[6] data and from an estimate of the mean effect of interstellar absorption given by Corlin.[13] The resulting 13 infrared magnitudes are then integrated to yield the total infrared magnitude (in the *K*- and *L*-bands) for each of the 48 areas. This final single magnitude is then converted to magnitudes per square degree. This flux is assigned to a "central point," to be defined later within each area. From these 48 data points per infrared band given in Table 3, the isophotes shown in Figs. 2 to 5 were estimated.

Results of the Computations, Isophotes

Figures 2 and 3 are the respective isophotes for the *K*- and *L*-bands plotted on galactic coordinates (l,b), and Figs. 4 and 5 are these same isophotes plotted on the Aitoff projection for equatorial coordinates (α,δ). Figures 4 and 5 were computed, respectively, from Figs. 2 and 3 by the nomograph in Fig. 6 for converting between galactic and equatorial coordinates. Figure 7 shows the isophotes for the visual magnitude computed from Charlier's numbers; Fig. 8, for comparison, presents a similar though more detailed (792 data points vs 48 here) plot from the Groningen compilation[14] as adapted from Roach and Megill.[5]

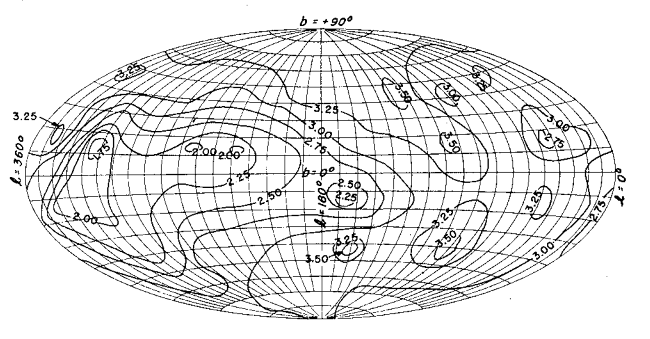

Fig. 3 Isophotes for the stellar sky in the *L*-band (3.6 μ) of Johnson; contour values are in magnitudes per square degree superimposed on Galactic coordinates.

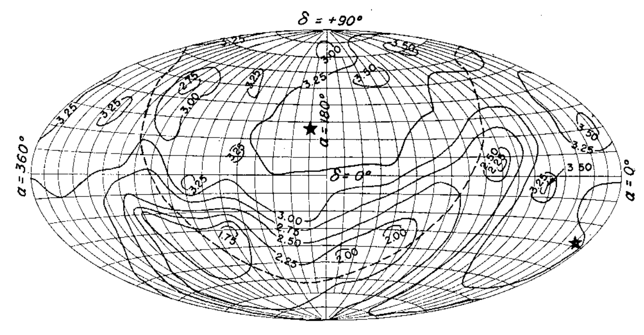

Fig. 5 Isophotes for the stellar sky in the *L*-band (3.6 μ) of Johnson; contour values are in magnitudes per square degree superimposed on equatorial coordinates.

Table 1 A rough comparison of the various sources of infrared radiation as compared to a 15.6-m-diam perfectly reflecting Echo-type satellite radiating as a blackbody at 232°K at 300 km[a]

Source	Radiation ratio, satellite:background			Remarks
	$3\ \mu$	$5\ \mu$	$10\ \mu$	
Solar radiation reflected by satellite	5×10^{-4}	1×10^{0}	2×10^{2}	Satellite albedo = 0.5, $T_{\odot} = 5740°K$
A single star:				Each star has been placed at a distance so that it appears to be of the first visual magnitude
*A*1V	3×10^{-2}	1×10^{1}	3×10^{4}	
*F*2V	1×10^{-2}	6×10^{0}	2×10^{4}	
*K*0III	2×10^{-1}	1×10^{0}	7×10^{2}	
*M*3III	1×10^{-6}	5×10^{-3}	2×10^{2}	
The sky at large, per deg,² away from the Milky Way	10^{-1}	10^{1}	10^{5}	Averaged over the whole sky without the Milky Way
The sky at large, per deg,² in the direction of the galactic center	$>10^{-5}$	$>10^{-2}$	$>10^{1}$	These figures represent a lower bound estimated by an integration of Schmidt's[1] model for all the galactic mass as *K*0III stars and no interstellar material
Zodiacal light, per deg²	10^{-3}	10^{-1}	10^{2}	These figures are for the bright part, only 10° away from the sun
Gegenschein, per deg²	$\gg 10^{-3}$	$\gg 10^{-1}$	$\gg 10^{2}$	The gegenschein is quite dim compared to the zodiacal light
Interstellar dust, per deg²	10^{36}	10^{23}	10^{6}	The interstellar dust is assumed to be at 50°K; some investigations indicate 20°K may be the case
Interstellar gas, *H*I region, per deg²	10^{14}	10^{7}	10^{-1}	The interstellar *H*I is assumed to be at 100°K
Interstellar gas, *H*II region, component *A*, for a single star:				Component *A*: the ionization of the general interstellar medium around hot stars, $N_H = 1$, $T = 10^4\ K$ for pure hydrogen and complete ionization; diameter of *H*II region = 1 k-parsec; emission due to free-free transition
*O*5	10^{-27}	10^{-8}	10^{10}	
*B*5I	10^{-24}	10^{-5}	10^{13}	
*A*0I	10^{-18}	10^{1}	10^{19}	
Interstellar gas, *H*II region, component *B*, for a single star:				Component *B*: a dense nebula (such as the Orion nebula) surrounding a hot star, $N_H = 10^4$, otherwise same as component *B*
*O*5				
*B*5I		10 times (*A*)		
*A*0I				

[a] All stars are assumed to radiate as blackbodies.

Johnson's *K*- and *L*-Band Data and the *K-V* and *L-V* Color Indices

Johnson's new system for infrared photometry contains four bands, *J*, *K*, *L*, and *M*, having, respectively, the following approximately square-topped transmission bands: 1.1–1.4 μ, 1.9–2.5 μ, 3.2–4.1 μ, and 4.4–5.5 μ. Johnson[6] has recently published *K* and *L* photoelectric magnitudes for 52 stars of various spectrum-luminosity types. A brief description of Johnson's photometer is given by him in Ref. 6.

Table 4 contains Johnson's *K* and *L* magnitudes for the range of spectrum-luminosity types. Columns 5 through 7 contain the corresponding (*U*, *B*, and *V*) magnitudes obtained from the references at the end of Table 4. Columns 8 and 9 contain the *K-V* and *L-V* color indexes, respectively.

Figures 9 and 10 contain, respectively, a plot of *K-V* and *L-V* color indexes vs spectral type. For stars of type *G* and later, the scatter and amount of data permitted only a rough separation of luminosity into two groups of luminosity classes I–III and IV–V. The smoothed data are represented by the solid line. This fit appears to be quite good for stars of type *G* and earlier. The only obvious anomalous data are those for eta Bootes. No speculation on the source of this anomaly was offered by Johnson; one will not be given here other than

Table 2 Mean fraction of stars in luminosity groups I–III and IV–V, P_s and $1\text{-}P_s$, respectively

Group	Spectral type						
	O	*B*	*A*	*F*	*G*	*K*	*M*,*N*
I–III	...	...	...	...	0.56	0.73	0.81
IV–V	...	...	...	...	0.44	0.27	0.19

that the star does not seem to be significantly reddened by interstellar material.

Table 5 indicates the computed interstellar reddening for the stars of Table 4 for which a B-V color index is given. The interstellar reddening A_V has been computed from the well-known formula

$$A_V = RE_{B\text{-}V} \tag{3}$$

where $E_{B\text{-}V}$ is the color excess given by

$$E_{B\text{-}V} = (M_B - M_V) - (B - V) \equiv CI - ci \tag{4}$$

and where R has been computed after the method of Blanco and Lennon[15] from the stellar spectral-energy distributions given by Johnson.[6] R varies from 3.2 for early stars to 3.8 for late-type giants. The intrinsic color index in Eq. (4) was taken from Fehrenbach.[16] Johnson[6] comments that the 52 stars, listed in Table 4, are essentially unreddened. This is born out by the data in Table 5, with few exceptions. Since the majority of these stars possess zero reddening, it may be assumed that the color indexes of Figs. 9 and 10 are the intrinsic color indexes.

The following computation shows that the deviations of the A_V values are less than the maximum expected error due to the impreciseness of the astronomical data needed to evaluate Eq. (3). If the linear terms of the Taylor series expansion of Eq. (3) are retained, then

$$\Delta A_V \cong \Delta R(\partial A_V/R\partial) + \Delta CI(\partial A_V/\partial CI) + \Delta ci(\partial A_V/\partial ci) \tag{5}$$

$$= E_{B\text{-}V}\Delta R + R\Delta CI - R\Delta ci \tag{6}$$

where ΔA_V is the error in the computed value of A_V resulting

Table 3 Integrated starlight within each of Charlier's 48 equal areas for the V (visual, 0.55 μ) band, and Johnson's K(2.2 μ) and L(3.6 μ) bands; the geometry of Charlier's areas; and Corlin's mean interstellar absorption, A_V

Charlier's area	Boundaries of Charlier's areas	Center of Charlier's areas	Integrated light, mag/deg^2			Corlin's interstellar absorption[a]
			V-band	K-band	L-band	
GA 1	$l = 90° \pm 90°, b = 78°22' \pm 11°38'$	$l = 90°, b = 73°37'$	6.31	2.52	3.29	0.80
2	$l = 270° \pm 90°, b = 78°22' \pm 11°38'$	$l = 270°, b = 73°37'$	5.79	2.64	3.41	0.05
GB 1	$l = 18° \pm 18°, b = 48°22' \pm 16°22'$	$l = 18°, b = 41°56'$	5.79	2.28	3.06	0.43
2	$l = 54° \pm 18°, b = 48°22' \pm 16°22'$	$l = 54°, b = 41°56'$	5.78	2.47	3.25	0.55
3	$l = 90° \pm 18°, b = 48°22' \pm 16°22'$	$l = 90°, b = 41°56'$	5.82	2.22	2.95	0.73
4	$l = 126° \pm 18°, b = 48°22' \pm 16°22'$	$l = 126°, b = 41°56'$	5.79	2.86	3.85	0.12
5	$l = 162° \pm 18°, b = 48°22' \pm 16°22'$	$l = 162°, b = 41°56'$	5.99	2.77	3.44	0.38
6	$l = 198° \pm 18°, b = 48°22' \pm 16°22'$	$l = 198°, b = 41°56'$	5.80	2.60	3.38	0.27
7	$l = 234° \pm 18°, b = 48°22' \pm 16°22'$	$l = 234°, b = 41°56'$	6.00	2.10	2.91	0.74
8	$l = 270° \pm 18°, b = 48°22' \pm 16°22'$	$l = 270°, b = 41°56'$	5.79	2.23	3.18	0.60
9	$l = 306° \pm 18°, b = 48°22' \pm 16°22'$	$l = 306°, b = 41°56'$	5.89	2.34	3.12	0.59
10	$l = 342° \pm 18°, b = 48°22' \pm 16°22'$	$l = 342°, b = 41°56'$	5.96	2.47	3.26	0.44
GC 1	$l = 15° \pm 15°, b = 15° \pm 15°$	$l = 15°, b = 14°29'$	5.51	2.28	2.99	0.4
2	$l = 45° \pm 15°, b = 15° \pm 15°$	$l = 45°, b = 14°29'$	5.10	1.98	2.73	0.52
3	$l = 75° \pm 15°, b = 15° \pm 15°$	$l = 75°, b = 14°29'$	5.52	2.37	3.12	0.37
4	$l = 105° \pm 15°, b = 15° \pm 15°$	$l = 105°, b = 14°29'$	5.73	2.83	3.56	0.31
5	$l = 135° \pm 15°, b = 15° \pm 15°$	$l = 135°, b = 14°29'$	5.37	2.47	3.19	0.22
6	$l = 165° \pm 15°, b = 15° \pm 15°$	$l = 165°, b = 14°29'$	5.23	2.24	3.00	0.26
7	$l = 195° \pm 15°, b = 15° \pm 15°$	$l = 195°, b = 14°29'$	5.20	1.88	2.49	0.80
8	$l = 225° \pm 15°, b = 15° \pm 15°$	$l = 225°, b = 14°29'$	5.37	1.29	1.97	1.25
9	$l = 255° \pm 15°, b = 15° \pm 15°$	$l = 255°, b = 14°29'$	5.09	1.02	1.92	0.80
10	$l = 285° \pm 15°, b = 15° \pm 15°$	$l = 285°, b = 14°29'$	4.83	1.70	2.43	0.65
11	$l = 315° \pm 15°, b = 15° \pm 15°$	$l = 315°, b = 14°29'$	5.09	0.58	1.53	1.38
12	$l = 345° \pm 15°, b = 15° \pm 15°$	$l = 345°, b = 14°29'$	5.68	2.75	3.43	0.32
GD 1	$l = 15° \pm 15°, b = -15° \pm 15°$	$l = 15°, b = -14°29'$	5.24	2.32	3.07	0.2
2	$l = 45° \pm 15°, b = -15° \pm 15°$	$l = 45°, b = -14°29'$	5.52	2.55	3.30	0.17
3	$l = 75° \pm 15°, b = -15° \pm 15°$	$l = 75°, b = -14°29'$	5.41	2.37	3.21	0.09
4	$l = 105° \pm 15°, b = -15° \pm 15°$	$l = 105°, b = -14°29'$	5.24	2.22	3.22	0.31
5	$l = 135° \pm 15°, b = -15° \pm 15°$	$l = 135°, b = -14°29'$	5.32	2.23	2.85	0.85
6	$l = 165° \pm 15°, b = -15° \pm 15°$	$l = 165°, b = -14°29'$	4.86	1.20	2.19	0.35
7	$l = 195° \pm 15°, b = -15° \pm 15°$	$l = 195°, b = -14°29'$	4.64	1.91	2.54	0.60
8	$l = 225° \pm 15°, b = -15° \pm 15°$	$l = 225°, b = -14°29$	4.80	1.75	2.44	(0.6)
9	$l = 255° \pm 15°, b = -15° \pm 15°$	$l = 255°, b = -14°29'$	4.78	1.64	2.34	(0.6)
10	$l = 285° \pm 15°, b = -15° \pm 15°$	$l = 285°, b = -14°29'$	4.90	1.45	2.17	0.62
11	$l = 315° \pm 15°, b = -15° \pm 15°$	$l = 315°, b = -14°29'$	4.94	1.08	1.87	1.12
12	$l = 345° \pm 15°, b = -15° \pm 15°$	$l = 345°, b = -14°29'$	5.22	1.17	1.88	1.36
GE 1	$l = 18° \pm 18°, b = -48°22' \pm 16°22'$	$l = 18°, b = -41°56'$	5.73	2.22	2.99	0.55
2	$l = 54° \pm 18°, b = -48°22' \pm 16°22'$	$l = 54°, b = -41°56'$	5.96	2.29	3.13	0.50
3	$l = 90° \pm 18°, b = -48°22' \pm 16°22'$	$l = 90°, b = -41°56'$	6.05	2.80	3.61	0.21
4	$l = 126° \pm 18°, b = -48°22' \pm 16°22'$	$l = 126°, b = -41°56'$	5.97	2.37	3.23	0.40
5	$l = 162° \pm 18°, b = -48°22' \pm 16°22'$	$l = 162°, b = -41°56'$	5.97	2.75	3.51	0.22
6	$l = 198° \pm 18°, b = -48°22' \pm 16°22'$	$l = 198°, b = -41°56'$	5.78	2.46	3.19	0.47
7	$l = 234° \pm 18°, b = -48°22' \pm 16°22'$	$l = 234°, b = -41°56'$	5.89	2.15	2.93	(0.7)
8	$l = 270° \pm 18°, b = -48°22' \pm 16°22'$	$l = 270°, b = -41°56'$	5.57	2.05	2.80	(0.7)
9	$l = 306° \pm 18°, b = -48°22' \pm 16°22'$	$l = 306°, b = -41°56'$	5.58	1.50	2.09	0.79
10	$l = 342° \pm 18°, b = -48°22' \pm 16°22'$	$l = 342°, b = -41°56'$	5.61	2.01	2.74	0.72
GF 1	$l = 90° \pm 90°, b = -78°22' \pm 11°38'$	$l = 90°, b = -73°37'$	6.04	2.08	2.86	0.83
2	$l = 270° \pm 90°, b = -78°22' \pm 11°38'$	$l = 270°, b = -73°37'$	6.05	2.18	3.15	0.92

[a] Numbers in parentheses have been estimated by two-way interpolation.

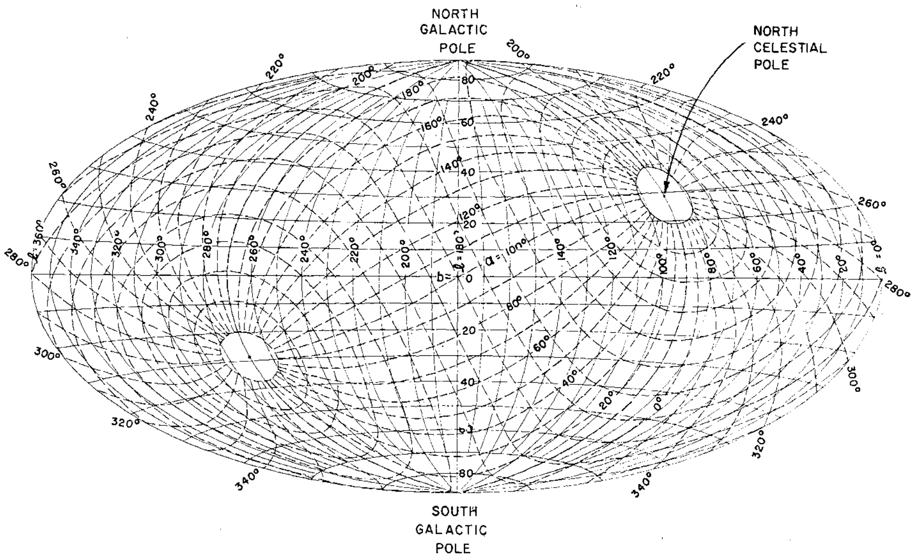

Fig .6 A nonograph for converting galactic coordinates (for the Ohlsson North Galactic Pole, i.e., at $\alpha = 190°$, $\delta = +28°$) to equatorial coordinates and vice versa for the epoch 1900.

from errors ΔR, ΔCI, and Δci in R, Ci, and ci, respectively. The impreciseness in R, CI, and ci is given by

$$\Delta R = \pm 0.2 \qquad \text{(Blanco and Lennon,[15] 1961)}$$
$$\Delta CI = \pm 0.1 \qquad \text{(Fehrenbach,[16] 1958)} \qquad (7)$$
$$\Delta ci = \pm 0.05$$

where Δci has been estimated from the scatter about the smooth curves in Figs. 9 and 10. When Eq. (7) is applied to Eq. (6), a maximum expected error is seen to be

$$\Delta|A_V|_{\max} \cong 0.2|E_{B\text{-}V}| + 3.2(0.1) + 3.2(0.05) \qquad (8)$$

$$= 0.2|E_{B\text{-}V}| + 0.48 \qquad (9)$$

$$< 0.2(0.20) + 0.48 = 0.52 \qquad (10)$$

where R has been taken as 3.2 and the maximum $|E_{B\text{-}V}| = 0.20$ has been used. It is seen that most of the A_V values in Table 5 are considerably less than 0.52.

This section and the previous section serve as introductory material for the mathematical description of the computational procedure that follows.

Mathematical Formalism of Computational Procedure and Interstellar Absorption

We shall first derive a relationship among an infrared apparent magnitude m_I and the intrinsic (I-V) color index of a given star, the effect of interstellar absorption A_V on this star, and its visual magnitude m_V. Throughout the remainder of this analysis, the visual magnitude of the "Henry Draper Catalogue" will be taken to be the same as the visual magnitude of Johnson and Morgan.[17] This assumption is quite justifiable with respect to the accuracy dealt with herein.[18]

In the absence of interstellar absorption, the apparent and intrinsic color indexes of a star must be identical:

$$m_I - m_V = M_I - M_V \qquad (11)$$

However, if the star is observed through interstellar material, Eq. (11) must be corrected to read

$$(m_I - A_I) - (m_V - A_V) = M_I - M_V \qquad (12)$$

where A_I and A_V are, respectively, the interstellar absorption in the arbitrary infrared I-band and in the V-band. Rearranging Eq. (12) yields

$$m_I = m_V - A_V + A_I + (M_I - M_V) \qquad (13)$$

which is the required relationship. This equation may be adapted to give the integrated infrared magnitude $\Sigma\, m_I$ to be defined below, within a given area of the sky, provided that the integration is restricted to a sufficiently narrow range of spectrum-luminosity types over which $(M_I - M_V)$ and $(A_I - A_V)$ do not vary significantly and provided that a suitable estimate of mean differential absorption $\langle A_I - A_V \rangle$ can be obtained:

$$\Sigma\, m_I \cong \Sigma\, m_V - \langle A_V - A_I \rangle + \langle M_I - M_V \rangle \qquad (14)$$

The spectrum-luminosity ranges used herein are dictated by Charlier's grouping, for spectral type, and by the luminosity division discernible in Figs. 9 and 10 (i.e., O, B–$B5$, ...). That for the intrinsic color index $M_I - M_V$ for $I = K, L$ does not vary too severely within each of our spectrum-luminosity groups from their values corresponding to the midspectral

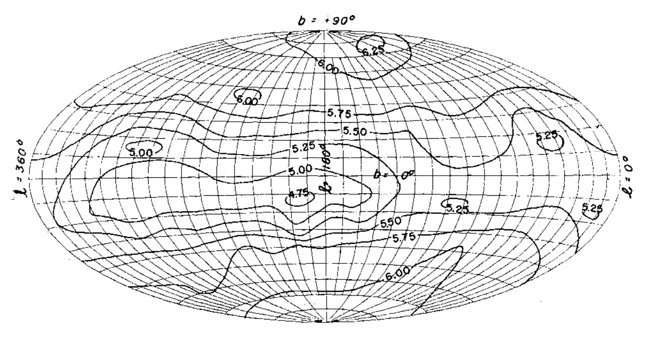

Fig. 7 Isophotes of the stellar sky in visual magnitudes per square degree superimposed on Galactic coordinates as computed from Charlier's data.

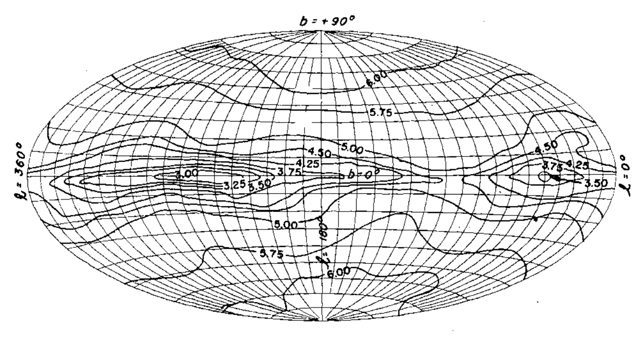

Fig. 8 Isophotes of the stellar sky in visual magnitudes per square degree as adapted from Roach and Megill.

types, can be seen from Figs. 9 and 10, where the largest variation (±0.7) is observed for Group *M*I–III. From Blanco's and Lennon's analysis,[15] it is apparent that A_V varies only slightly within our groups, provided that the color excess E_{B-V} is maintained constant.

However, in reality, E_{B-V} is far from constant in any area since the stars are at varying distances and hence experience varying amounts of reddening. Therefore, $\langle A_I - A_V \rangle$ for a limited spectrum-luminosity range must be a mean over distance or, more precisely, over optical depth of the interstellar material. No such calculation has been attempted here; instead Corlin's[13] mean values for each of Charlier's areas have been assumed to be representative for all spectral types. Corlin's values are listed in the last column in Table 3. Corlin's mean absorptions were computed from observations of stars having measured trigonometric parallaxes and hence represent the interstellar extinction of only the nearby stars having parallaxes greater than 0.″02. The use of Corlin's data for our purpose can only be partially justified as follows: 1) Corlin's values are based on observations of the nearby stars and, hence, should be sufficiently representative of these stars; this is fortunate since all the nearby stars are comparatively bright and nearly independent of spectral type, and will therefore be substantial contributors to the infrared inte-

Table 4 Infrared and visual magnitudes[a]

Star name	*Sp*[h]	*K*	*K-L*	*V*	*B-V*	*U-B*	*K-V*	*L-V*
α And	(*B8p*)	2.46	...	2.12	...	...	0.34	...[b]
β Cas	*F*2 IV	1.41	−0.05	2.25	0.35	0.09	−0.84	−0.79
χ Peg	*M*2 III	0.67	0.04	4.80	1.58	1.92	−4.13	−4.17
α Ari	*K*2 III	−0.61	0.12	1.99	1.15	1.12	−2.60	−2.72[f]
ρ Per	*M*4	−1.90	...	3.4	...	...	...	...[e,j]
HR 483	*G*2 V	3.39	...	5.1	0.63	0.12	−1.7	...[d]
α Per	*F*5 I*b*	0.59	0.03	1.82	...	...	−1.23	−1.25[b]
ϵ Eri	*K*2 V	1.72	0.20	3.73	0.89	0.57	−2.01	−2.21
α Eri	*K*3 III	−0.91	...	0.55	...	...	−1.46	...[b, i]
δ Tau	*K*O III	1.50	...	3.73	0.98	0.84	−2.23	...
α Tau	*K*5 III	−2.82	0.22	0.78	1.51	1.81	−3.60	−3.82[f]
α Aur	*G*8 III + *F*	−1.78	0.06	0.09	0.80	0.43	−1.87	−1.93
β Ori	*B*8 I*a*	0.19	−0.10	0.08	−0.03	−0.69	0.11	0.21
γ Ori	*B*2 III	2.12	...	1.64	−0.23	−0.87	0.48	...
α Lep	*F*O I*b*	1.99	...	2.56	0.21	0.22	−0.57	...
α Ori	*M*2-*M*3 I*ab*	−4.16	...	0.7	...	...	−4.9	...[b, e]
β Aur	*A*2 IV	1.94	...	1.90	0.03	0.06	0.04	...
γ Gem	*A*O IV	1.92	0.17	1.95	0.00	0.01	−0.03	−0.20
α CMa	*A*1 V	−1.38	0.05	−1.47	0.01	−0.08	0.09	0.04
α Gem	*Am* + *A*1 V	1.46	...	1.59	0.04	0.02	−0.13	...
α CMi	*F*5 IV	−0.69	0.07	0.34	0.40	−0.01	−1.03	−1.10
β Gem	*K*O III	−1.09	0.13	1.15	1.00	0.84	−2.24	−2.37[f]
ζ Pup	*O*5*f*	3.07	...	2.25	−0.26	−1.11	0.82	...
β Cnc	*K*4 III	0.15	0.09	3.53	1.48	1.77	−3.38	−3.47
α Leo	*B*7 V	1.66	−0.05	1.33	−0.12	−0.38	0.33	−0.33
γ Leo	*K*O III*p*	−0.65	0.14	2.21	...	...	−2.86	−3.00[b]
μ UMa	*M*O III	−0.79	0.12	3.0	...	...	−3.8	−3.9
α UMa	*K*O III	−0.59	0.15	1.79	1.06	0.90	−2.38	−2.53
Lal 21185	*M*2 V	3.79	0.23	7.47	1.51	1.26	−3.68	−3.91
γ Vir	...	2.02	...	2.74	0.36	−0.05	−0.72	...[c]
β Com	*G*O V	2.77	0.22	4.30	0.56	0.05	−1.53	−1.75
Barnard's Star	*M*5 V	4.88		9.54	1.74	1.29	−4.66	...
α Vir	*B*2 V	1.61	−0.21	0.98	−0.26	−0.94	0.63	−0.84
2 Cen	*M*6 III	−1.65	0.20	4.2	...	...	−5.9	−6.1[h]
η Boo	*G*O IV	−1.42	−0.01	2.70	0.59	0.20	−4.12	−4.11
θ Cen	*K*O III-IV	−0.23	0.13	1.98	...	...	−2.21	−2.34
HR 5299	*M*4 III	−0.26	0.28	5.13	...	...	−5.39	−5.67[h]
α Boo	*K*2 III*p*	−3.03	0.13	−0.06	1.23	1.26	−2.97	−3.10[f]
α Ser	*K*3 III	0.13	0.11	2.66	1.16	1.24	−2.53	−2.64
ϵ CrB	*K*3 III	1.35	0.12	4.15	1.23	1.28	−2.80	−2.92
α Sco	Compound	−3.83	0.32	0.92	1.84	1.30	−4.75	−5.07[c]
ζ Her	*G*O IV	1.32	−0.16	2.82	0.64	0.21	−1.50	−1.34
β Oph	*K*2 III	0.23	0.13	2.77	1.16	1.24	−2.54	−2.67
γ Dra	*K*5 III	−1.25	0.19	2.22	1.52	1.86	−3.47	−3.66
α Lyr	*A*O V	0.02	0.04	0.04	0.00	−0.01	−0.02	−0.06
β Cyg	Compound	0.21	0.06	...	...	...	...	...[c]
σ Dra	*K*O V	2.90	0.23	4.68	0.79	0.39	−1.78	−2.01
61 Cyg *B*	*K*5 *V*	2.76	0.01	6.02	1.38	1.23	−3.26	−3.27
α Aql	*A*7 IV, V	0.25	0.06	0.77	0.22	0.08	−0.52	−0.58
α Cyg	*A*2 I*a*	0.93	0.08	1.26	0.09	−0.25	−0.33	−0.41
ϵ Cyg	*K*O III	0.17	0.10	2.45	1.03	0.87	−2.28	−2.38[f]
β Peg	*M*2 II-III	−2.10	0.25	2.52	...	...	−4.62	−4.87[b]

[a] *K* and *L* magnitudes taken from Ref. 6; *U*, *B*, and *V* (Johnson's and Morgan's system) are taken from Ref. 17, 18, 31–33.
[b] *V* taken from Ref. 2.
[c] Magnitudes given for double system.
[d] *V* taken from the *m* column of Ref. 34.
[e] Variable.
[f] Standard.
[g] Spectrum-luminosity classifications same as in Ref. 6 unless otherwise indicated.
[h] Harvard classification converted to the MKK system (Morgan, Keenan, and Kellman[35]) by Johnson.[6]
[i] Spectrum-luminosity classification taken from Ref. 2.
[j] *Sp* and *V* taken from *Skalnate Pleso Atlas*, Czechoslovakia.

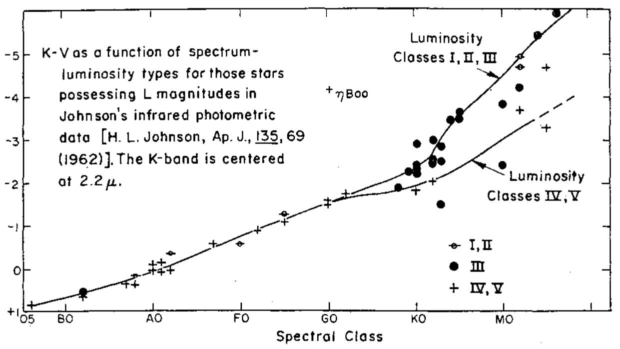

Fig. 9 $K - V$ **vs spectral type.**

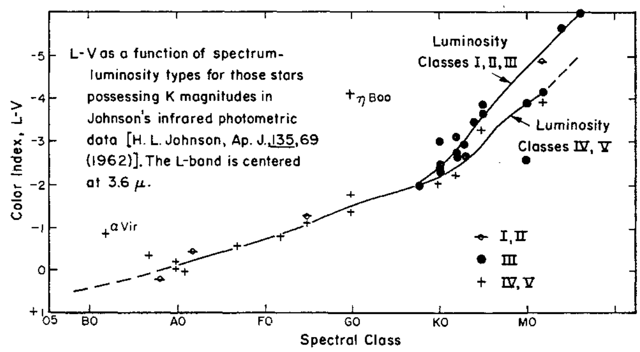

Fig. 10 $L - V$ **vs spectral type.**

grated starlight even for the hottest, the O and B, stars. 2) The more distant hot and intrinsically luminous stars will contribute little to the integrated infrared starlight by virtue of the comparatively (to cool stars) small amount of their energy emitted in the infrared wavelengths and the inverse-square radiation relationship. A crude integration, within small solid angles directed toward the Milky Way, of Schmidt's[1] model indicates that, within these areas, which are dense in cool stars (the intrinsically greatest infrared emitters), the contribution to the infrared integrated starlight is predominantly from the distant stars (except from the far other side of the galaxy). The mixture of stellar types at various positions along these directions was estimated from Allen.[2] The mass and luminosity calibration of the spectrum-luminosity types were also taken from Allen. Therefore, our adapted mean visual absorption along and near the galactic equator and especially in the region of the galactic center must be in error by several magnitudes, and hence we shall have underestimated the integrated infrared starlight by a similar amount in these regions. Stebbins and Whitford[19,24] and Dufay et al.[20] indicate that A_V for the galactic center may be 10 magnitudes and certain dark clouds might absorb 20 magnitudes. The cumulative area of the dark clouds, with the exception of the neighborhood in the direction of the galactic center, is small, and hopefully the dark clouds need not be considered in our use of the mean absorption.[13] However, the general interstellar material in the plane of the Milky Way must be considered in the construction of any detailed plot of infrared isophotes. By necessity, the area over which our integration has been performed is large (859 deg^2) in comparison to the fraction of one of these areas, which is occupied by general Milky Way obscuring material. Hence, the isophotes in Figs. 9 and 10 are only mildly affected by this gross error in A_V, since the error occurs in an area small in comparison to the total area over which the integration was performed. The neglect of the exceptionally large A_V in the region of the galactic center ($l = 325°$, b 0°) is reflected by the lack of detail in this region of Figs. 9 and 10. In summary, Corlin's values, for our purposes, are probably adequate for the sky without the Milky Way, and in the Milky Way their inaccuracies would seem not to affect our isophotes greatly because of coarseness.

Within the spirit of our simple approximation for the infrared sky, the use of Corlin's values conveniently permits a comparatively quick estimate of our problem of approximating the infrared sky. The computation of the infrared sky may

Table 5 Interstellar reddening of Johnson's stars

Star name	E_{B-V}	$A_V = RE_{B-V}$	Star name	E_{B-V}	$A_V = RE_{B-V}$
α And	...	...	β Com	0.04	0.13
β Cas	0.05	0.15	Barnard's Star	−0.05	−0.18
χ Peg	−0.01	−0.03	α Vir	0.02	0.06
α Ari	0.01	0.03	2 Cen	...	...
ρ Per	...	...	η Boo	0.01	0.03
HR 483	0.01	0.03	θ Cen	...	...
α Per	...	...	HR 4299	...	...
ε Eri	0.04	0.12	α Boo	−0.07	−0.25
α Eri	...	...	α Ser	0.14	0.50
δ Tau	0.03	0.09	ε CrB	0.07	0.25
α Tau	0.01	0.03	α Sco	...	...
α Aur	0.15	0.54	ζ Her	0.04	0.12
β Ori	0.02	0.06	β Oph	0.00	0.00
γ Ori	−0.04	−0.12	γ Dra	0.00	0.00
α Lep	0.02	0.06	α Lyr	0.00	0.00
α Ori	...	...	β Cyg	...	...
β Aur	0.04	0.12	σ Dra	0.03	0.09
γ Gem	0.00	0.00	61 Cyg B	−0.20	−0.64
α CMa	0.04	0.12	α Aql	−0.03	0.09
α Gem	0.01	0.03	α Cyg	−0.09	−0.28
α CMi	0.04	0.12	ε Cyg	0.02	0.06
β Gem	0.01	0.03	β Peg	...	...
ζ Pup	−0.06	−0.18	...	...	...
β Cnc	−0.07	−0.25	...	...	...
α Leo	−0.01	−0.03	...	...	...
γ Leo	...	...	...	...	...
μ UMa	...	...	...	...	...
α UMa	−0.05	−0.18	...	...	...
Lal 21185	−0.03	−0.11	...	...	...
γ Vir	...	...	...	...	...

be performed to its ultimate accuracy by considering each star individually and applying a suitable procedure to account for both the visual and infrared interstellar absorption. However, such a method must, of necessity, depend on a large high-speed computing machine and on a punched-card compilation of stellar data like the one of NASA.[22, 23]

Returning to Eq. (14), a considerable simplification ca be achieved if A_I is taken to be zero, or at least small in comparison to A_V. This can be seen from Whitford's[24] interstellar absorption curve. Equation (14) then becomes

$$\Sigma\, m_I \cong \Sigma\, m_V - \langle A_V \rangle + \langle M_I - M_V \rangle \tag{15}$$

where the M_V, $\langle A_V \rangle$, and $\langle M_I - M_V \rangle$ are to be found, respectively, in Charlier's compilation, Corlin's values given in Table 3, and Figs. 9 or 10 computed from Johnson's data.

The operation of Σ can be simply deduced from the definition of the astronomical magnitude

$$m_i - m_{i,0} = 2.5 \log_{10}(I_{i,0}/I_i) \tag{16}$$

where m_i and $m_{i,0}$ are, respectively, the magnitudes corresponding to the intensities I_i and $I_{i,0}$, for the ith band. $I_{i,0}$ will be considered as a calibration intensity having the standard magnitude $m_{i,0}$. On solving for I_i, Eq. (16) becomes

$$I_i = I_{i,0}\, 10^{0.4(m_{i,0} - m_i)} \tag{17}$$

The integrated intensity for an arbitrary grouping of stars is

$$\mathcal{I} = \Sigma\, I_i = I_{i,0}\, 10^{0.4 m_{i,0}}\, \Sigma\, 10^{-0.4 m_i} \tag{18}$$

by Eq. (17), where the summation is over the stars in the group. By Eq. (16) the integrated magnitude is

$$\Sigma\, m_i = m_{i,0} + 2.5 \log_{10}(I_{i,0}/\mathcal{I}_i) \tag{19}$$

$$= 2.5 \log_{10}[\Sigma 10^{-0.4 m_i}] \tag{20}$$

In terms of star count numbers, the integrated magnitude of N stars [having a spectrum-luminosity range S, magnitude interval $m_i \pm \frac{1}{2}\Delta m_i$, and within an area at (l,b)] is

$$\sum_N m_i = 2.5 \log_{10} [N_{S,m_i}(l,b) 10^{-0.4 m_i}] \tag{21}$$

$$= 2.5 \log_{10}[N_{S,m_i}(l,b)] - m_i \tag{22}$$

For Charlier's compilation and our further division into luminosity groups, 13 integrated magnitudes (for our groups O, B-$B5$, . . .) for each area are computed by a subsequent integration over magnitude (i.e., over Charlier's intervals -1 ± 0.5, 0 ± 0.5, . . .):

$$\sum_{m_V} \left(\sum_N m_V\right) = \sum_{m_V} [2.5 \log_{10} N_A - m_V] \tag{23}$$

which, by Eq. (20), becomes

$$\sum_{m_V} \sum_N m_V = 2.5 \log_{10} \left[\sum_{m_V} 10^{-\log_{10} N_A + 0.4 m_V}\right] \tag{24}$$

These 13 resulting twice-integrated magnitudes (or representative stars) yield 13 corresponding infrared magnitudes by Eq. (15):

$$\sum m_I \equiv \sum_{N,m_V} m_I = \sum_{m_V} \sum_N m_V - \langle A_V \rangle + \langle M_I - M_V \rangle \tag{25}$$

The resulting $13 \Sigma m_I$'s may then be integrated to give the total integrated infrared starlight $\mathfrak{M}_I$ within a given area:

$$\mathfrak{M}_I = \sum_S \sum_{N,m_V} m_I \tag{26}$$

$$\cong \sum_S \left[\sum_{m_V} \sum_N m_V - \langle A_V \rangle + \langle M_I - M_V \rangle\right] \tag{27}$$

$$= 2.5 \log_{10} \left\{\sum_S {}^{-0.4} \left[\sum_{m_V} \sum_N m_V - \langle A_V \rangle + \langle M_i - M_V \rangle\right]\right\} \tag{28}$$

$$= \langle A_V \rangle + 2.5 \log_{10} \times \left\{\sum_S 10^{-\log_{10}\left[\sum_{m_V} 10^{-\log_{10} N_A + 0.4 m_V}\right] + 0.4 \langle M_I - M_V \rangle}\right\} \tag{29}$$

where Eqs. (27) and (28) follow from Eqs. (20) and (25), and Eq. (29) from Eq. (24), and the assumed constancy of $\langle A_V \rangle$ with respect to spectrum-luminosity type within any area.

These seemingly difficult computations for integrated magnitudes have been reduced to desk-calculator operations by using Gyllenberg's[25] tables. The value obtained from Eq. (29) is for an area $\Omega = 859.44$ deg², and can be converted to a flux $d\mathfrak{M}_I/d\omega$ in magnitude/deg² by applying the definition of an astronomical magnitude as given by Eq. (16):

$$(d\mathfrak{M}_I/d\omega) = \mathfrak{M}_I + 2.5 \log_{10}(859.44/1) = \mathfrak{M}_I + 7.3356 \text{ mag/deg}^2 \tag{30}$$

The flux computed in this way from Charlier's compilation for Johnson's K- and L-bands are given in columns 5 and 6 of Table 3. Column 4 gives similar figures for the V-band, where

$$\mathfrak{M}_V = \sum_S \sum_{m_V} \sum_N m_V \tag{31}$$

It is from these data in columns 4 through 6 of Table 3 that the isophotes of Figs. 7, 2, and 3, respectively, have been estimated. The points in each area which have been chosen to represent the flux are given in the third column of Table 3 labeled "Center of Charlier's Areas." Here "center" is meant to be the center of moments b_c on the midlongitude (the first number of column 3) of each area. By Pappus' theorem, the following formula for b_c is easily derived:

$$\frac{\frac{1}{2}\Delta b + \{b_c - [(\pi/2) - b_m]\}}{\frac{1}{2}\Delta b - \{b_c - [(\pi/2) - b_m]\}} = \frac{\sin\{\frac{1}{2}b_c + \frac{1}{2}[(\pi/2) - b_m] + \frac{1}{2}\Delta b\}}{\sin\{\frac{1}{2}b_c + \frac{1}{2}[(\pi/2) - b_m] - \frac{1}{2}\Delta b\}} \tag{32}$$

where b_m and $\frac{1}{2}\Delta b$ are, respectively, the midlatitude and half-height of an area. The values $b_m \pm \frac{1}{2}\Delta b$ for each of Charlier's areas are given by the second number in column 2 of Table 3 and were used as the inputs for numerically solving Eq. (32).

The isophotes in Figs. 2, 3, and 7 have been computed by filling in a sufficient number of points between the 48 data points by successive two-way quadratic interpolations and then by making a final interpolation along a linear approximation of the gradient to obtain quarter-magnitude contours. The small number of data points from which these isophotes were computed places their detailed features in suspect. The inability of our data points to produce accurate detail, especially toward the galactic center and along the galactic equator, is demonstrated by comparing Fig. 7 for our 48 data points with Roach and Megill[5] for 792 data points for Fig. 8. However, the general features of Fig. 8 are preserved in our Fig. 7. The most obvious differences are the "peaks" in Fig. 8 which have been smoothed out over a much larger area in our computations. (Roach's and Megill's areas are 40 deg² near the galactic equator as compared to our 859 deg².)

The total visual magnitude for the whole sky has been estimated by Seares et al.[26] to be 1092 stars of visual magnitude +1.0 or an integrated figure of −6.50. This compares favorably with our figure

$$(\mathfrak{M}_V)_{\text{total}} = \sum_{\Omega(l,b)} \sum_S \sum_{m_V} \sum_N m_V = -6.09 \tag{33}$$

since if the difference $|-6.50-(-6.09)|$ is distributed in some manner (i.e., does not occur in one small region) throughout the whole sky, the difference per square degree at any point should be quite small.

The total K and L magnitudes for the whole sky are

$$(\mathfrak{M}_K)_{\text{total}} = \sum_{\Omega(l,b)} \mathfrak{M}_K = -9.56 \tag{34}$$

$$(\mathfrak{M}_L)_{\text{total}} = \sum_{\Omega(l,b)} \mathfrak{M}_L = -8.78 \tag{35}$$

Conversion of Magnitude to Intensity

To relate magnitude to intensity, we proceed as follows. If Eq. (16) is rearranged, then we have

$$m_i = (m_{i,0} + 2.5 \log_{10} I_{i,0}) - 2.5 \log_{10} I_i \tag{36}$$

$$= C_i - 2.5 \log_{10} I_i \tag{37}$$

If j represents a band distinct from i, then

$$m_i - m_j = (C_i - C_j) - 2.5 \log_{10}(I_i/I_j) \tag{38}$$

For a normal unreddened main-sequence AOV star, $m_i - m_j$ will be defined to be zero. This is in fact the case for $m_K - m_V$ and $m_L - m_V$.[6] Therefore,

$$C_j = C_i - 2.5 \log_{10}(I_i/I_j)_{AOV} \tag{39}$$

where by Eq. (37),

$$C_i = m_{i,*} + 2.5 \log_{10} I_{i,*} \tag{40}$$

where $m_{i,*}$ is some standard magnitude in band i corresponding to the intensity $I_{i,*}$. Therefore, on combining Eqs. (39) and (40),

$$C_j = (m_{i,*} + 2.5 \log_{10} I_{i,*}) - 2.5 \log_{10}(I_i/I_j)_{AOV} \tag{41}$$

By Eq. (37) and by replacing i by j,

$$\log_{10} I_j = 0.4(C_j - m_j) \tag{42}$$

and hence, by Eq. (41),

$$I_j = [10^{0.4m_{i,*} + \log_{10}(I_{i,*}I_{j,AOV}/I_{i,AOV})}]10^{-0.4m_j} \tag{43}$$

$$= k_j\, 10^{-0.4m_j} \tag{44}$$

If $m_{i,*}$ and $I_{i,*}$ are evaluated for our AOV standard, k_j becomes

$$k_j = 10^{0.4m_{i,AOV} + \log_{10} I_{j,AOV}} \tag{45}$$

$$= [10^{0.4m_{i,AOV}}]I_{j,AOV} \tag{46}$$

Furthermore, if α Lyra is taken as our AOV standard, i.e.,

$$m_{i,AOV} \equiv V_{\alpha\,\text{Lyra}} = +0.04 \tag{47}$$

then by Eq. (46), for j alternately replaced by K and L,

$$k_j = [10^{0.4(0.04)}]I_{j,\alpha\,\text{Lyra}} = 1.04\, I_{j,\alpha\,\text{Lyra}} \tag{48}$$

$$k_K = 1.04\, I_{K,\alpha\,\text{Lyra}} \tag{49}$$

$$k_L = 1.04\, I_{L,\alpha\,\text{Lyra}} \tag{50}$$

Hence, by Eqs. (43) and (48), the intensity in the j-band can be computed from the magnitude m_j if the j intensity for α Lyra is known. Unfortunately, Johnson has not yet published these figures for his K- and L-bands, but does give a sufficient description of his photometer to permit an estimate of these intensities. Allen (Ref. 2, pp. 23 and 24) gives the intensity for a star having $m_V = 0$ outside the earth's atmosphere as

$$I_{(m_V=0)} = 2.43 \times 10^{-10}\ \text{lm-m}^{-2} = 2.43 \times 10^{-10}(1/60\pi)\sigma(2042)^4 \times 10^{-4} = 1.40 \times 10^{-7}\ \text{w-cm}^{-2} \tag{51}$$

For the case of α Lyra

$$I_{V,\alpha\,\text{Lyra}} = I_{(m_V=0.04)} = 1.40 \times 10^{-7} \log_{10}^{-1}(0.04/2.5) = 1.35 \times 10^{-7}\ \text{w-cm}^{-2} \tag{52}$$

If $S(\lambda)$, $\phi_V(\lambda)$, and $\phi_K(\lambda)$ are, respectively, the stellar spectral energy distribution and V- and K-band response functions, then the V and K magnitudes are given by

$$V = -2.5 \log_{10}\left[\frac{\int S(\lambda)\phi_V(\lambda)d\lambda}{I_{V,0}}\right] + V_0 \tag{53}$$

$$K = -2.5 \log_{10}\left[\frac{\int S(\lambda)\phi_K(\lambda)d\lambda}{I_{K,0}}\right] + K_0 \tag{54}$$

If the standard intensities $I_{V,0}$ and $I_{K,0}$ are those of α Lyra, then Eqs. (53) and (54) yield

$$I_{K,\alpha\,\text{Lyra}} = \left[\frac{\int S_{\alpha\,\text{Lyra}}(\lambda)\phi_K(\lambda)d\lambda}{\int S_{\alpha\,\text{Lyra}}(\lambda)\phi_V(\lambda)d\lambda}\right] I_{V,\alpha\,\text{Lyra}} \tag{55}$$

and similarly for the L-band,

$$I_{L,\alpha\,\text{Lyra}} = \left[\frac{\int S_{\alpha\,\text{Lyra}}(\lambda)\phi_L(\lambda)d\lambda}{\int S_{\alpha\,\text{Lyra}}(\lambda)\phi_V(\lambda)d\lambda}\right] I_{V,\alpha\,\text{Lyra}} \tag{56}$$

These are independent of the units of $S_{\alpha\,\text{Lyra}}(\lambda)$. Reference 18 gives $\phi_V(\lambda)$. $\phi_K(\lambda)$ and $\phi_L(\lambda)$ have been estimated, to within a constant factor, by assuming the filter transmissivities $\sim$0.70 and to be square topped over Johnson's[6] published approximate band limits, using two aluminum reflections and the reflectivity given by Allen (Ref. 2, p. 23) and the relative sensitivity of InSb (77 K) given by Texas Instruments.[27] $S_{\alpha\,\text{Lyra}}(\lambda)$ may be estimated to within a constant factor for wavelengths $\lambda_V = 0.55$ and greater by the blackbody formula for the effective temperature of an AOV star:

$$S_{\alpha\,\text{Lyra}}(\lambda) \cong (\text{const})_1\, B_\lambda(T_{AOV}),\ T_{AOV} = T_{\alpha\,\text{Lyra}} = 10{,}500\ \text{K} \tag{57}$$

This is seen to be nearly true by comparing a first approximation for the continuum of a theoretical stellar atmosphere by an AOV star with the blackbody curve for the effective temperature and same total luminosity.[28, 29] When the indicated numerical integrations are performed, Eqs. (52, 55, and 56) give

$$\begin{aligned} I_{K,\alpha\,\text{Lyra}} &\approx 0.47\ (\text{const})_2\, I_{V,\alpha\,\text{Lyra}} \\ I_{L,\alpha\,\text{Lyra}} &\approx 0.92\ (\text{const})_2\, I_{V,\alpha\,\text{Lyra}} \end{aligned} \tag{58}$$

The forementioned constant occurring in Eq. (58) is the factor needed to convert the relative units of ϕ_K and ϕ_L to those of ϕ_V. A detailed calculation of this factor will not be attempted; however, an estimate will be made by comparing "figures of merit," after Jones[30] for 1P21-RCA:V-band and the InSb-Texas Instruments:K,L-band sensors. This comparison approximates the constant in Eq. (58) to be 2.3×10^{-3}. Therefore, our estimates for the K and L intensities of α Lyra are, by Eqs. (52) and (58),

$$\begin{aligned} I_{K,\alpha\,\text{Lyra}} &\sim 1.5 \times 10^{-10}\ \text{w-cm}^{-2} \\ I_{L,\alpha\,\text{Lyra}} &\sim 2.8 \times 10^{-10}\ \text{w-cm}^{-2} \end{aligned} \tag{59}$$

It must be emphasized that the numbers in Eq. (58), and even more so in Eq. (59), are only estimates subject to a number of errors; the exact shape of the K and L response functions in consistent units with the V response function, Eq. (57), is based on a first approximation for a continuum and the stellar-atmospheric absorption features superimposed on the continuum in the K- and L-bands.

If the integrated magnitude or intensity over an area other than one of Charlier's is desired, the following formulas, applied to our isophotes, can be used. If $I_i = I_i(l,b)$ is a continuous intensity per unit area within some closed region, then by Eq. (17)

$$I_i(l,b)dldb = [I_{i,0}10^{0.4m_{i,0}}]10^{0.4m_i(l,b)}dldb \tag{60}$$

where $m_i(l,b)$ is the corresponding magnitude per square degree. Therefore, the integrated intensity is

$$\mathcal{I}_i = \iint_{b\,l} I_i(l,b)dldb = [I_{i,0}\ 10^{0.4m_{i,0}}] \iint_{b\,l} 10^{-0.4m_i(l,b)}\ dldb \tag{61}$$

and the corresponding integrated magnitude is

$$\mathfrak{M}_i = 2.5 \log_{10} \left[\iint_{b\,l} 10^{-0.4m_i(l,b)}\ dldb \right] \tag{62}$$

By Eqs. (46) and (61),

$$\mathcal{I}_K = k_K \iint_{b\,l} 10^{-0.4m_K(l,b)}\ dldb$$
$$\mathcal{I}_L = k_L \iint_{b\,l} 10^{-0.4m_L(l,b)}\ dldb \tag{63}$$

and by Eqs. (50) and (59), the following estimate is offered:

$$\mathcal{I}_K \sim 1.6 \times 10^{-10} \iint_{b\,l} 10^{-0.4m_K(l,b)}\ dldb$$
$$\mathcal{I}_L \sim 2.9 \times 10^{-10} \iint_{b\,l} 10^{-0.4m_L(l,b)}\ dldb \tag{64}$$

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